2901. Split the fraction up, and consider it as a pair of geometric series.

2902. (a) Set the first derivative to zero. Solve to find the SPs, and evaluate the second derivative.
(b) The equation is a quadratic in $x^{\frac{1}{3}}$.
(c) Consider division by zero.
(d) Put it all together!

2903. This is, in one sense, an obvious result. Obvious results are often best proved by contradiction.

2904. Sketch the curve carefully, then set up two definite integrals to calculate the area.

2905. (a) This is a one-tailed test.
(b) Give your answer in set notation.
(c) Work out whether there is sufficient evidence to reject H_0. Give a conclusion in context.
(d) Consider whether the normal distribution can be applied in such circumstances.

2906. Find the y coordinates of the intersections, and set up a single definite integral with respect to y.

2907. Consider NII for each monkey, paying particular attention to the value of the tension.

2908. Determine the number of roots of each factor. The square on the first factor doesn't affect things.

2909. This is a form of a log law.

2910. Consider a function which is self-inverse.

2911. (a) Substitute $t = 0$, and find R.
(b) Set $R = R_0$ and solve.
(c) Rearrange to make R the subject, set the first derivative to zero and solve.
(d) The rate of change of response is maximised when the second derivative of R is zero.

2912. (a) Consider the ratio c_{n+1}/c_n.
(b) Write $a_n = ar^{n-1}$ and $b_n = br^{n-1}$.

2913. Take the RHS as an expression, and express it as a single fraction.

2914. Write the sum out longhand, and simplify.

2915. Consider the decimal as $x = 0.a_1a_2a_3...a_n$, where a_i represents the ith digit.

2916. It's correct!

2917. (a) Set the numerator to zero.
(b) Set up a definite integral between $x = 1$ and $x = 21$. Use the substitution $u = 1 + 3x$.

2918. Show, using symmetry, that the centres lie at the corners of an equilateral triangle.

2919. Rearrange to $x^2 = y + 1$, substitute and simplify.

2920. The placement of the first bishop matters: split the cases up by distance from the edge of the board. Conditioning (multiplying probabilities) is easier than combinatorics (counting outcomes).

2921. Set up a generic parabola $y = ax^2 + bx + c$. Find its derivative, and substitute into the DE. Once you've subbed in, you need an *identity* in x.

2922. Factorise the expression on the LHS, and consider the signs of the factors.

2923. Use the change of base formula.

2924. (a) Consider a vertical asymptote.
(b) Consider a point where the third derivative is also zero.

2925. Remember that a definite integral calculates the *signed* area. In (c), use harmonic form.

2926. Using Pythagoras, set up an expression for the squared length of each chord. Equate them and solve, choosing the appropriate root.

2927. Differentiate implicitly wrt x. Then set $\frac{dy}{dx}$ to 2, and show that the resulting equation has no roots.

2928. Draw a force diagram, and resolve perpendicular and parallel to the plane.

2929. Consider the number of orders of 9 objects. Then visualise a long list of these orders, grouping by position:

1,2,3	4,5,6	7,8,9
1,2,3	4,5,6	7,9,8
1,2,3	4,5,6	8,7,9
...	...	...

Find the number of orders within each partitioned group of three. These are overcounting factors, which you then need to divide by.

2930. Add the two equations and rearrange to $v+u = ...$ Also subtract the two equations and rearrange to $v - u = ...$ Multiply these expressions together to reach $v^2 - u^2 = ...$

2931. Write the sum out longhand, then expand the brackets. Most of the terms will cancel, leaving a simple answer. You could try setting $n = 2$ to begin with, to see what's going on.

2932. (a) Differentiate.
(b) Show that both $\sin(\ln x)$ and $\cos(\ln x)$ are +ve for $x \in (1, 2]$.
(c) Consider the meaning of the sign of the second derivative, as a direction of curvature.
(d) Show that the area below the tangent line is $\frac{1}{2}$. This overestimates the integral.

2933. Start with the RHS. Firstly, use

$$\tan\theta \equiv \frac{\sin\theta}{\cos\theta}.$$

Then multiply top and bottom of the fraction by $\cos^2\theta$, and use two more identities, one on the top and one on the bottom.

2934. The first counter can be placed wlog. Then place the second, then the third.

2935. The error is with the chain rule.

2936. (a) Using NIII, the total downwards force on the pulley is $R = T_1 + T_2$.
(b) Set up an equation of motion for each block. Use part (a) to eliminate the tensions.

2937. Prove this by contradiction.

2938. Rearrange each equation to make $\frac{y}{x}$ the subject (considering the possibility $x = 0$). Equate these, and you'll get a quadratic in e^x.

2939. (a) Find the equation of the tangent.
(b) This is about the intersection of a tangent and the curve to which it is tangent.

2940. Sketch the four boundary equations. The region you're looking for is a square.

2941. List in such a way that you know it is complete, e.g. by number of white beads.

2942. Differentiate implicitly using the chain rule, and rearrange to make $\frac{dy}{dx}$ the subject. You'll need to multiply by $\sqrt{x+y}$ at some point.

2943. (a) The four curved sections make a full circle.
(b) Consider symmetry.
(c) Draw a force diagram for one of the upper logs.

2944. Sketch the mod graph. You don't need to find the intersections with the unit circle, just the angle subtended by the successful sector. To find this, work out the gradient(s) of the mod graph. The probability is then the area of the successful region divided by the area of the circle.

2945. Prove this by contradiction, looking at the number of roots of the equation $f(x) = g(x)$.

2946. This is a standard quotable result: integration by inspection via the reverse chain rule. The reason, in this case, is differentiation by the chain rule.

2947. Sketch the mod graph and the circle carefully. In this case, because the mod graph is not a smooth curve, it is possible that the shortest distance doesn't lie along a normal.

2948. (a) Set up an integral with respect to t, whose t limits are 0 and ∞. Enact the substitution.
(b) The form for the partial fractions is

$$\frac{1}{u(u-1)} \equiv \frac{A}{u} + \frac{B}{u-1}.$$

Find A and B, then integrate.

2949. Consider rolling the m-sided die first.

2950. This is not true.

2951. Use the first Pythagorean identity to form a cubic.

2952. Assume, for a contradiction, that $\log_3 5 = p/q$, for $p, q \in \mathbb{N}$. Rewrite as an index statement. Raise both sides to the power q, and find a contradiction based on prime factors.

2953. (a) Sketch the function (you could use $f(x) \propto x^3$) marking the given area. Remember: a definite integral calculates the *signed* area.
(b) Use the fact of rotational symmetry to write the integrand as $2\,f(2x)$. Then integrate by substitution, with $u = 2x$.

2954. Use the chain rule to find $\dfrac{dx_n}{dx_1}$. Then integrate.

2955. (a) If you can't figure this out, try part (b) first: that calculation and numerical result (which isn't 4%) form as good an explanation as any.
(b) Draw a tree diagram, conditioning on the type of car. Then restrict the possibility space to cars *identified* as saloons.

2956. You don't need to use calculus here. Find the range of the denominator first; it is a quadratic in $\ln x$, so complete the square.

2957. You can simplify this entire problem significantly by translating it by $-c\mathbf{j}$. Set the problem up with $y = x^3 - x$, whose point of inflection is at O.

Find the equation of the normal, and then find its intersections with the curve. Evaluate the area of the region for $x \geq 0$ with a definite integral. The curve has rotational symmetry, so the areas will automatically be equal to each other.

2958. Using the vertex, express the quadratic in completed-square form, and solve $y = 0$.

2959. (a) Show that two of the lines have perpendicular gradients.
(b) Solve simultaneously.
(c) Identify the relevant lines, then calculate the angle of inclination (above/below the positive x direction) of each.
(d) Use right-angled trigonometry on triangle T.
(e) Call the shortest length d. Use the result from (c) to find an expression for the area in terms of d. Set up an equation and solve for d.

2960. Consider the number of different arrangements of PIZ$_1$Z$_2$AZ$_3$Z$_4$ first.

2961. Factorise fully and consider the nature (that is to say, the multiplicity) of the roots.

2962. Express $a_2 = b_2$ and $a_3 = b_5$ in terms of the first term of both sequences a, the common ratio r and the common difference d. Eliminate d to produce a single equation in a and r. Factorise and solve.

2963. Consider the domain of definition, the nature of any roots, and the presence and type of any SPs.

2964. Consider the rearrangements of four items, with E and A forming a single item EA.

2965. This is a quadratic in $\sqrt{x}$.

2966. For $X \sim B(6, 0.27)$, find the mean. Then calculate the probabilities of the outcomes either side of the mean. Explain how they disprove the claim.

2967. Consider the constant term 33, which has factors 3 and 11. Look for two quadratics with these as constant terms:

$$\begin{aligned} &x^4 + 2x^3 + 15x^2 + 14x + 33 \\ \equiv\ &(x^2 + ax + 3)(x^2 + bx + 11). \end{aligned}$$

Equate coefficients of x and of x^3 for simultaneous equations in a and b. Solve, and then consider Δ.

2968. (a) Draw a force diagram, with the ladder on the point of slipping, so friction at $F_{\text{max}} = \mu R$. Resolve vertically and horizontally, and take moments about the base of the ladder.
(b) As θ tends to zero, the value of $\cot\theta$ tends to infinity. What does this mean physically?

2969. This is integration by inspection: the reverse chain rule. Remember that "write down" means that there is little or no calculation needed: you can go straight to the answer.

2970. Write the equation as a polynomial in 2^x. The factor theorem tells you that $(2^x - 3)$ must be a factor.

2971. (a) Quote the standard formula.
(b) Set up a point (p, p^2) on the parabola. By equating the squared lengths, find the value of p such that the chords to $(0, 0)$ and $(1, 1)$ are of equal length. Then calculate the area using this value.

2972. Use the parametric differentiation formula

$$\frac{dy}{dx} = \frac{\frac{dy}{dt}}{\frac{dx}{dt}}.$$

Equate $\dfrac{dy}{dx}$ to the gradient of the vector.

2973. Show that all of the stationary points of the curve have positive y values.

2974. There are only two successful outcomes.

2975. (a) Reflect $y = e^x$ in the line $y = x$.
(b) Differentiate.
(c) Compare the equation you find in (b) to $y = x$: you need to show that $y = \ln x$ and $y = x$ do not intersect.

2976. Find the mean of the arithmetic progression, which must be the interior angle of a regular hexagon. Then put upper and lower bounds on the common difference d:

① upper: the smallest angle is 0,
② lower: the hexagon is regular.

Consider whether these are attainable or not.

2977. This is possible if the means $(\bar{x}, \bar{y})$ of the samples differ significantly, when compared to their spreads.

2978. This is a geometric series. Work out the common ratio and the number of terms.

2979. Consider the graph $y = \sqrt[4]{x}$ as a version of the graph $y = \sqrt{x}$. They are related in the same way as $y = x^4$ is related to $y = x^2$. Having sketched $y = \sqrt[4]{x}$, apply an input transformation.

2980. (a) Set up horizontal and vertical *suvat* for the flight, and eliminate t.
(b) Using the second Pythagorean trig identity, express the equation in (a) as a quadratic in $\tan\theta$, and calculate its discriminant.

2981. Use a combinatorics argument: consider the total number of arrangements, and the number which are successful.

2982. Find the tangent to $y = \ln x$ at $x = 1$.

2983. For constant jerk, we know that $\dddot{x} = j$, where $\dddot{x}$ is the third derivative of x with respect to t and j is a constant. Integrate this equation three times, introducing a constant of integration each time. You should end up with a cubic in t; it has much in common with $x = x_0 + v_0 t + \frac{1}{2}at^2$.

2984. Express the area of an equilateral triangle in terms of its side length. The graph of A against t will be given by this formula over a certain domain, and by 0 outside it.

2985. Use log rules. In (b) and (c), raise the base and input to the same power.

2986. Find the equation of the chord: $y - y_1 = m(x - x_1)$. Set up an equation for intersections with the curve. Look for a repeated factor.

2987. Each equation can be considered as a quadratic in t, so use the quadratic formula.

2988. Simplify the RHS as an expression.

2989. (a) Find A as a function of p. Let $p \to \infty$.
(b) Set $A = 0$ and solve.
(c) Set $\frac{dA}{dp} = 0$ for optimisation.
(d) Use all of the above.

2990. Show that the function $f(x) = x^2 - e^{x^2} + 2$ has a stationary point at $x = 0$, and explain why this breaks the Newton-Raphson method.

2991. (a) Evaluate the derivatives of e^x.
(b) Find the derivatives of g and equate them to the values found in (a).
(c) Note that the two should follow each other closely around $x = 0$.

2992. Call the perpendicular side lengths a and b, and set up simultaneous equations.

2993. Consider this graphically: think about the nature of the intersection of the graphs at $x = k$.

2994. (a) Set the denominator to zero.
(b) By writing the top in terms of the bottom, or by long division, express the improper fraction properly, i.e. as

$$y = Ax + B + \frac{C}{5 - x}.$$

Then consider the behaviour as $x \to \pm\infty$.

2995. (a) Solve simultaneously for intersections.
(b) Find $y_{\text{line}} - y_{\text{curve}}$ in terms of x and c.
(c) Using (a) and (b), compare the area of the shaded region with the area of a parallelogram.

2996. Draw a tree diagram, conditioned on the choice of coin. The restrict the possibility space to those outcomes in which the coin shows heads.

2997. Use log rules. You can convert the first logarithm to base p by taking the square root of both base and input.

2998. Assuming limiting equilibrium, find the tension in the string. Then draw clear force diagrams for the two stacked blocks, separating them to include the interaction between them.

2999. For partial fractions, set up

$$\frac{3x+3}{x^2+3x} \equiv \frac{A}{x+3} + \frac{B}{x}$$
$$\Longrightarrow 3x + 3 \equiv Ax + B(x+3).$$

Equate coefficients or else substitute values. Once you've got the partial fractions, integrate. Use log rules to combine the resulting logarithms.

3000. Find the first derivative using the quotient rule.

——— End of Volume III ———